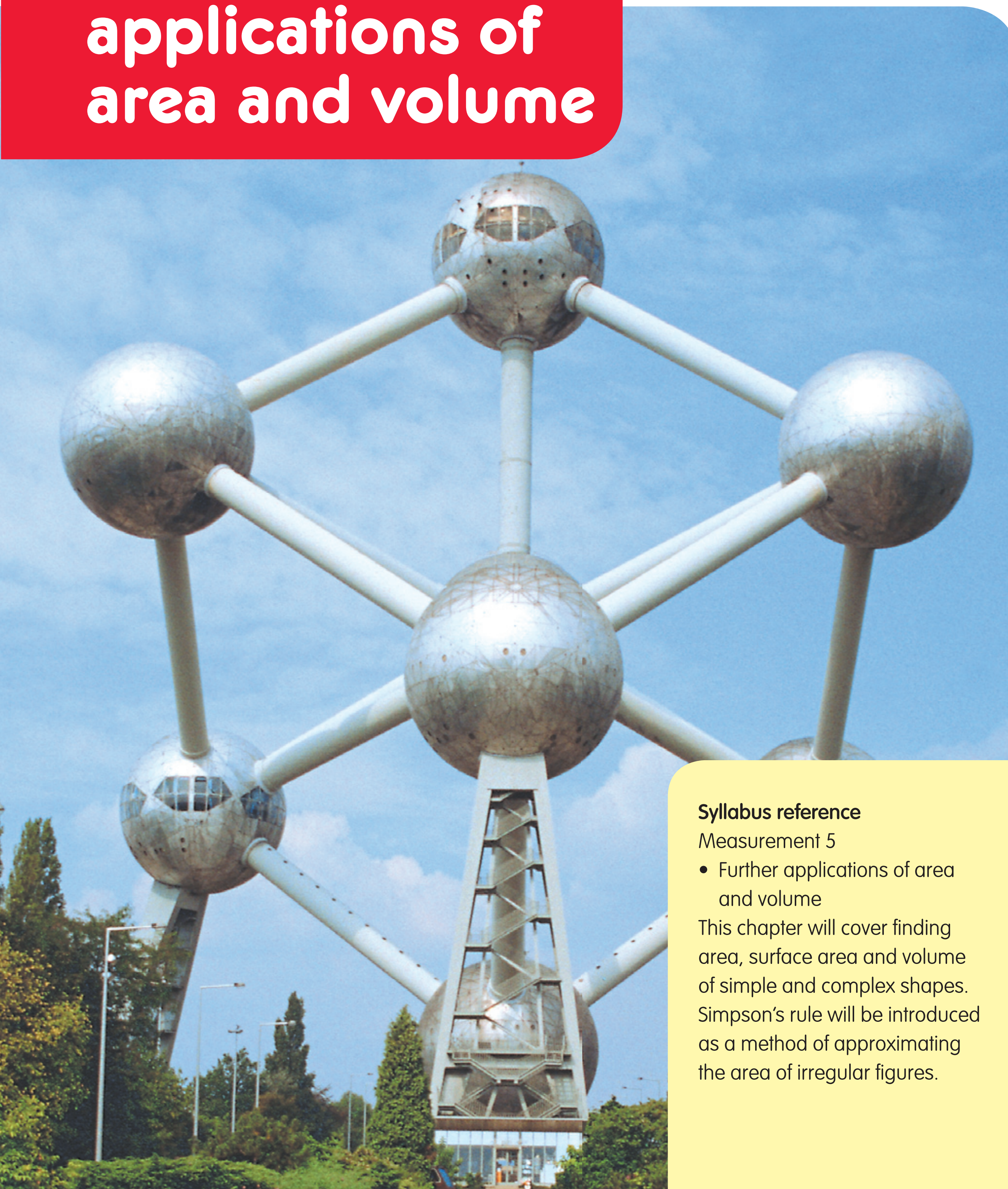
2

Further applications of area and volume



Syllabus reference
Measurement 5
- Further applications of area and volume

This chapter will cover finding area, surface area and volume of simple and complex shapes. Simpson's rule will be introduced as a method of approximating the area of irregular figures.

ARE YOU READY?

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Area of a circle

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the eBookPLUS or ask your teacher for a copy.

Area of a circle

1 Find the area of a circle with:
a radius 4 cm **b** radius 19.6 cm **c** diameter 9 cm **d** diameter 19.7 cm.

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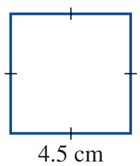
Areas of squares, rectangles and triangles

Areas of squares, rectangles and triangles

2 Find the area of each of the following.

a


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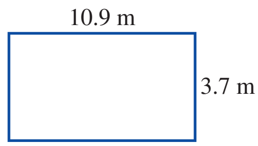
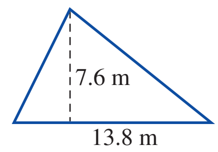


c


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Volume of cubes and rectangular prisms

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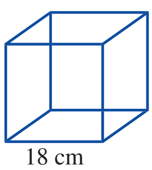
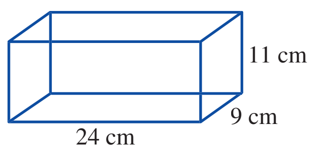
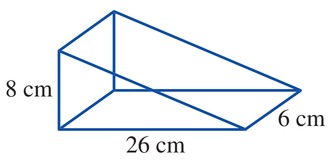
Volume of triangular prisms

Volume of cubes and rectangular prisms (3a, 3b); Volume of triangular prisms (3c)

3 Find the volume of:

a


b


c


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Volume of cylinders

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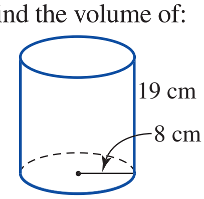
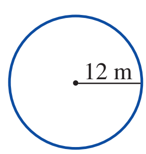
Volume of a sphere

Volume of cylinders (4a); Volume of a sphere (4b)

4 Find the volume of:

a


b


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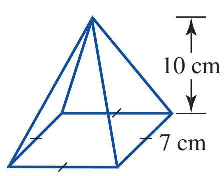
Volume of a pyramid

Volume of a pyramid

5 Find the volume of:



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Error in linear measurement

Error in linear measurement

6 For each of the following linear measurements, state the limits between which the true limits actually lie.
a 15 cm (measured correct to the nearest centimetre)
b 8.3 m (measured correct to 1 decimal place)
c 4800 km (measured correct to the nearest 100 km)

2A Area of parts of the circle

From previous work you should know that the area of a circle can be calculated using the formula:

$$A = \pi r^2$$

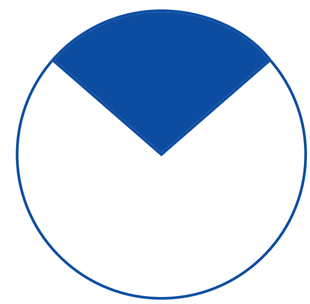
WORKED EXAMPLE 1

Calculate the area of a circle with a radius of 7.2 cm. Give your answer correct to 2 decimal places.

THINK	WRITE
1 Write the formula.	$A = \pi r^2$
2 Substitute for the radius.	$= \pi \times (7.2)^2$
3 Calculate the area.	$= 162.86\ \text{cm}^2$

A **sector** is the part of a circle between two radii as shown on the right.

To calculate the area of a sector we find the fraction of the circle formed by the sector. For example, a semicircle is half of a circle and so the area of a semicircle is half the area of a full circle. A **quadrant** is a quarter of a circle and so the area is quarter that of a full circle.



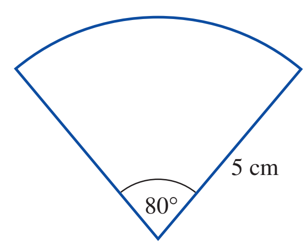
For other sectors the area is calculated by using the angle between the radii as a fraction of 360° and then multiplying by the area of the full circle. This can be written using the formula:

$$A = \frac{\theta}{360}\pi r^2$$

where θ is the angle between the two radii.

WORKED EXAMPLE 2

Calculate the area of the sector drawn on the right.
Give your answer correct to 1 decimal place.



THINK	WRITE
1 Write the formula.	$A = \frac{\theta}{360}\pi r^2$
2 Substitute for θ and r.	$= \frac{80}{360} \times \pi \times 5^2$
3 Calculate the area.	$= 17.5\ \text{cm}^2$

An **annulus** is the area between two circles that have the same centre (i.e. concentric circles). The area of an annulus is found by subtracting the area of the smaller circle from the area of the larger circle. This translates to the formula $A = \pi(R^2 - r^2)$, where R is the radius of the outer circle and r is the radius of the inner circle.

WORKED EXAMPLE 3

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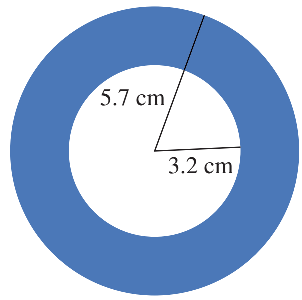
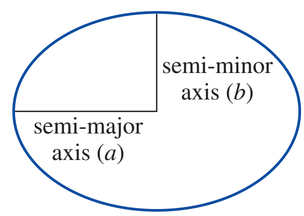
Worked example 3

Calculate the area of the annulus on the right. Give your answer correct to 1 decimal place.



THINK	WRITE
1 Write the formula.	$A = \pi(R^2 - r^2)$
2 Substitute $R = 5.7$ and $r = 3.2$.	$= \pi(5.7^2 - 3.2^2)$
3 Calculate.	$= 69.9\text{ cm}^2$

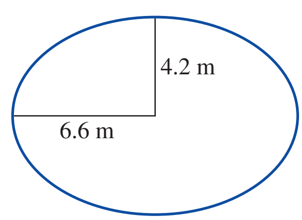
An **ellipse** is an oval shape and therefore does not have a constant radius. The greatest distance from the centre of the ellipse to the circumference is called the semi-major axis, a, while the smallest distance is called the semi-minor axis, b, as shown in the figure on the right. The area of an ellipse is calculated using the formula, found on the formula sheet:



$$A = \pi ab$$

WORKED EXAMPLE 4

Calculate the area of the ellipse drawn on the right. Give your answer correct to 2 decimal places.



THINK	WRITE
1 Write the formula.	$A = \pi ab$
2 Substitute for the values of a and b.	$= \pi \times 6.6 \times 4.2$
3 Calculate the area.	$= 87.08\text{ m}^2$

REMEMBER

1. The area of a circle is found using the formula $A = \pi r^2$.
2. The area of a sector can be found by multiplying the area of a full circle by the fraction of the circle given by the angle in the sector. You can use the formula $A = \frac{\theta}{360} \times \pi r^2$.
3. An annulus is the area between two concentric circles. The area is found by using the formula $A = \pi(R^2 - r^2)$, where R is the radius of the outer circle and r is the radius of the inner circle.
4. An ellipse is an oval shape. The area is calculated using the formula $A = \pi ab$, where a is the length of the semi-major axis and b is the length of the semi-minor axis.

EXERCISE 2A Area of parts of the circle

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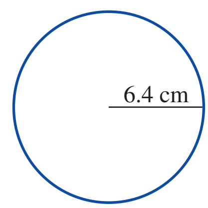
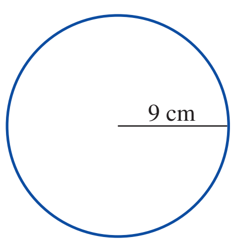
Area of a circle

1 WE1 Calculate the area of the circle drawn on the right, correct to 1 decimal place.

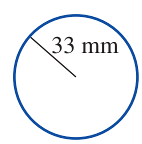


2 Calculate the area of each of the circles drawn below, correct to 2 decimal places.

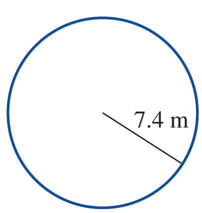
a



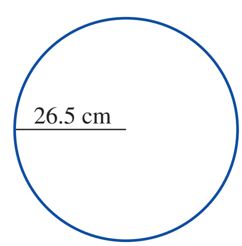
b



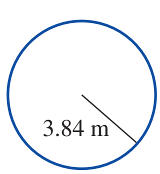
c



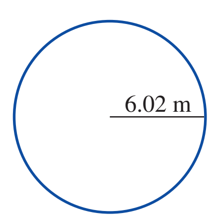
d



e

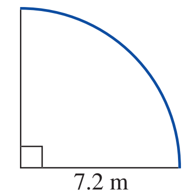


f



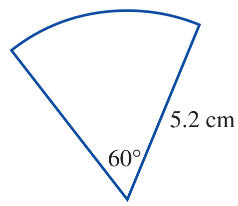
3 Calculate the area of a circle that has a diameter of 15 m. Give your answer correct to 1 decimal place.

4 WE2 Calculate the area of the sector drawn on the right. Give your answer correct to 1 decimal place.

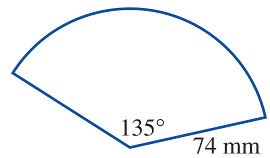


5 Calculate the area of each of the sectors drawn below. Give each answer correct to 2 decimal places.

a



b



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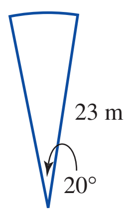
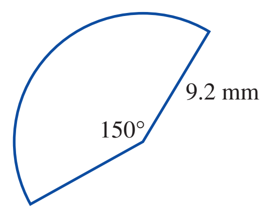


d



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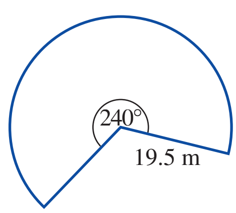
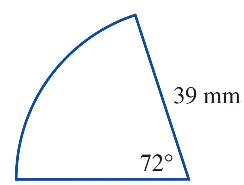
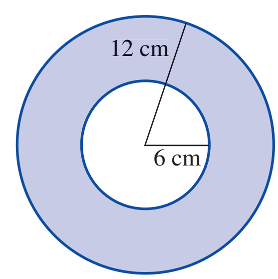


f



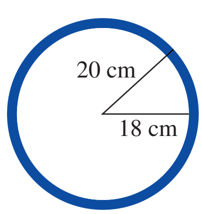
6 Calculate, correct to 1 decimal place, the area of a semicircle with a diameter of 45.9 cm.

7 WE3 Calculate the area of the annulus shown at right, correct to 1 decimal place.

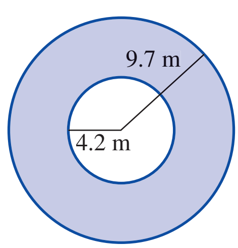


8 Calculate the area of each annulus drawn below, correct to 3 significant figures.

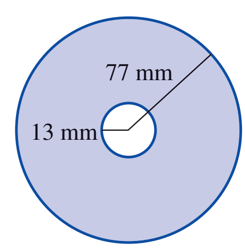
a



b



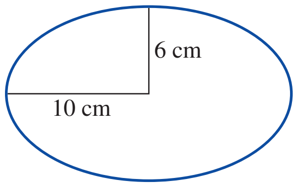
c



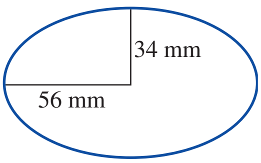
9 A circular garden of diameter 5 m is to have concrete laid around it. The concrete is to be 1 m wide.

a What is the radius of the garden?
b What is the radius of the concrete circle?
c Calculate the area of the concrete, correct to 1 decimal place.

10 WE4 Calculate the area of the ellipse drawn on the right, correct to 1 decimal place.



11 Calculate the area of each of the ellipses drawn below. Give each answer correct to the nearest whole number.

a


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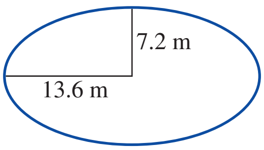
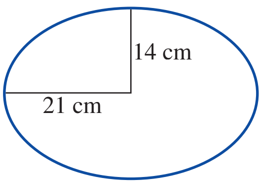
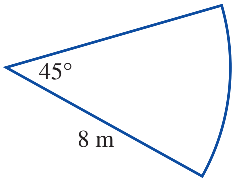


c


12 MC The area of a circle with a diameter of 4.8 m is closest to:

A $15\ m^2$ B $18\ m^2$ C $36\ m^2$ D $72\ m^2$

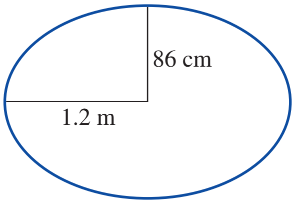
13 MC Which of the following calculations will give the area of the sector shown on the right?



A $\frac{1}{8} \times \pi \times 4^2$ B $\frac{1}{8} \times \pi \times 8^2$

C $\frac{1}{4} \times \pi \times 4^2$ D $\frac{1}{4} \times \pi \times 8^2$

14 MC The area of the ellipse drawn on the right is closest to:



A $32\,400\ cm^2$ B $324\ m^2$
C $5900\ cm^2$ D $59\ m^2$

Further development

15 A circular area is pegged out and has a diameter of 10 m.

a Calculate the area of this circle, correct to 1 decimal place.
b A garden is to be dug which is 3 m wide around the area that has been pegged out. Calculate the area of the garden to be dug. Give your answer correct to 1 decimal place.
c In the garden a sector with an angle of 75° at the centre is to be used to plant roses. Calculate the area of the rose garden, correct to 1 decimal place.

16 A circle has a diameter of 20 cm.

a Calculate the area of this circle, correct to 2 decimal places.
b An ellipse is drawn such that the radius of the circle forms the semi-major axis. The semi-minor axis is to have a length equal to half the radius of the circle. Calculate the length of the semi-minor axis.
c Calculate the area of the ellipse, correct to 2 decimal places.

17 The area in front of a building is rectangular in shape, measuring 50 metres by 15 metres. At night a security light scans the area. The security light, if positioned vertically against the wall at ground level, illuminates an area of the wall that is a sector of a circle of radius 15 m and has an angle of 60° at the centre.

- **a** What is the area of the rectangular frontage?
- **b** What is the area that is illuminated at any one time by the security light? Give your answer correct to 1 decimal place.
- **c** What percentage of the frontage is illuminated at any one time? Give your answer correct to 1 decimal place.

18 A circular island is in the centre of a circular lake such that the surface of the water in the lake forms an annulus. The radius of the lake is 10 m greater than the radius of the island.

- **a** Given that the island has a radius of 20 m, find the area of the surface of the water. Give your answer correct to the nearest square metre.
- **b** Tori claims that the surface area of the lake will remain the same regardless of the two radii as long as the difference of 10 metres remains unchanged. Is Tori correct? Use calculations to justify your response.

19 An ellipse has an area of 50 cm^2 and the length of one semi-axis is 5 cm.

- **a** Find, correct to 1 decimal place, the length of the other semi-axis.
- **b** Which of the two axes is the semi-major axis and what is its length? What is the length of the semi-minor axis?

20 An arc of length 1 cm stands on a circle of radius 1 cm. Find the size of the angle subtended at the centre correct to the nearest degree.

21 An ellipse has semi-major and semi-minor axes of 6 cm and 4 cm respectively.

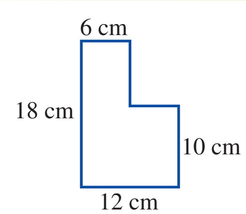
- **a** Calculate the area of the ellipse correct to 1 decimal place.
- **b** Frank claims that the ellipse will have the same area as a circle of radius 5 cm. Is Frank correct?
- **c** Find the radius of the circle that has the same area as the ellipse. (Answer correct to 1 decimal place.)

2B Area of composite shapes

A composite shape is a shape that is made up of two or more regular shapes. The area of a composite shape is found by splitting the area into two or more regular shapes and calculating the area of each separately before adding them together. In many cases it will be necessary to calculate the length of a missing side before calculating the area. There will sometimes be more than one way to split the composite shape.

WORKED EXAMPLE 5

Find the area of the figure at right.

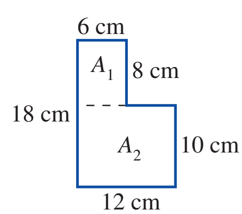


THIS

THINK

WRITE

1 Copy the diagram and divide the shape into two rectangles.



2	Calculate the length of the missing side in rectangle 1. (Write this on the diagram.)	$18 - 10 = 8\text{ cm}$
3	Calculate the area of rectangle 1.	$A_1 = 6 \times 8$ $= 48\text{ cm}^2$
4	Calculate the area of rectangle 2.	$A_2 = 10 \times 12$ $= 120\text{ cm}^2$
5	Add together the two areas.	$\text{Area} = 48 + 120$ $= 168\text{ cm}^2$

Composite areas that involve triangles may require you to also make a calculation using Pythagoras' theorem.

WORKED EXAMPLE 6

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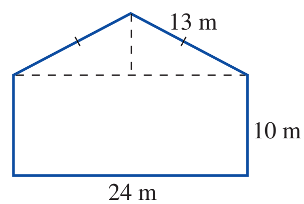
Worked example 6

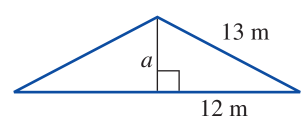
Find the area of the figure on the right.



THINK

WRITE

1 Draw the triangle at the top and cut the isosceles triangle in half.



2 Calculate the perpendicular height using Pythagoras' theorem.

$$\begin{aligned} a^2 &= c^2 - b^2 \\ &= 13^2 - 12^2 \\ &= 169 - 144 \\ &= 25 \\ a &= \sqrt{25} \\ &= 5\text{ m} \end{aligned}$$

3 Calculate the area of the triangle.

$$\begin{aligned} A &= \tfrac{1}{2} \times 24 \times 5 \\ &= 60\text{ m}^2 \end{aligned}$$

4 Calculate the area of the rectangle.

$$\begin{aligned} A &= 24 \times 10 \\ &= 240\text{ m}^2 \end{aligned}$$

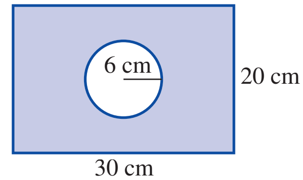
5 Add the two areas together.

$$\begin{aligned} \text{Area} &= 60 + 240 \\ &= 300\text{ m}^2 \end{aligned}$$

Composite areas can also be calculated by using subtraction rather than addition. In these cases we calculate the larger area and subtract the smaller area in the same way as we did with annuluses in the previous section.

WORKED EXAMPLE 7

Find the shaded area in the figure on the right.



THINK	WRITE
1 Calculate the area of the rectangle.	$A = 30 \times 20$ $= 600\text{ cm}^2$
2 Calculate the area of the circle.	$A = \pi \times 6^2$ $= 113.1\text{ cm}^2$
3 Subtract the areas.	$\text{Area} = 600 - 113.1$ $= 486.9\text{ cm}^2$

REMEMBER

1. To find the area of any composite figure, divide the shape into smaller regular shapes and calculate each area separately.
2. You may have to use Pythagoras' theorem to find missing pieces of information.
3. Check if the best way to solve the question is by adding two areas or by subtracting one area from the other to find the remaining area.

EXERCISE 2B Area of composite shapes

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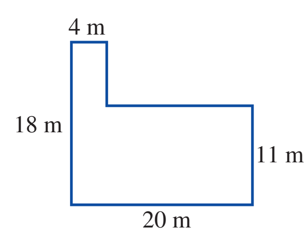
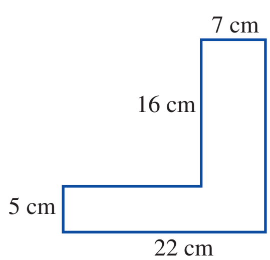
Areas of squares, rectangles and triangles

1 WE5 Copy the figure on the right into your workbook and calculate its area by dividing it into two rectangles.

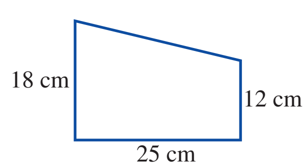


2 Find the area of each of the figures below. Where necessary, give your answer correct to 1 decimal place.

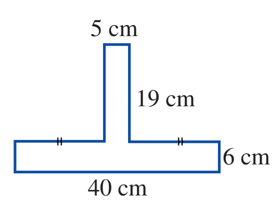
a



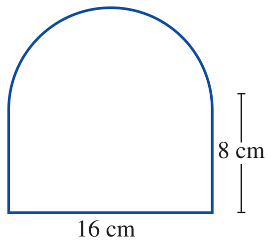
b



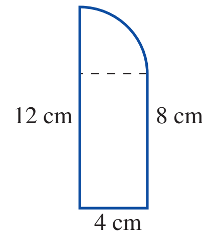
c



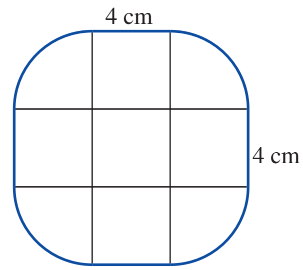
d



e



f



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SkillSHEET 2.3
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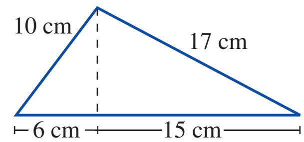
Using Pythagoras' theorem

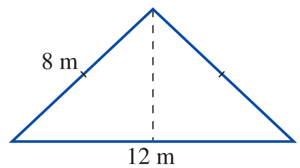
3 Look at the triangle on the right.

a Use Pythagoras' theorem to find the perpendicular height of the triangle.

b Calculate the area of the triangle.



4 Below is an isosceles triangle.



a Use Pythagoras' theorem to find the perpendicular height of the triangle, correct to 1 decimal place.

b Calculate the area of the triangle.

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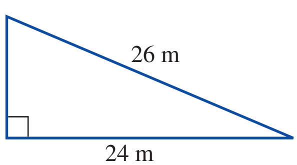
Digital doc
Excel spreadsheet
doc-1306
Pythagoras

eBook*plus*

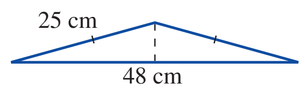
Digital doc
GC program — Casio
doc-1307
Mensuration

5 Calculate the area of each of the triangles below. Where necessary, give your answer correct to 1 decimal place.

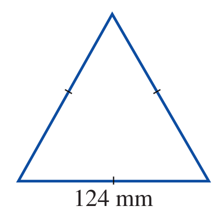
a



b

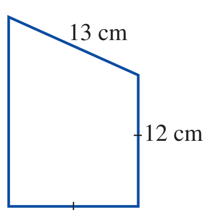


c

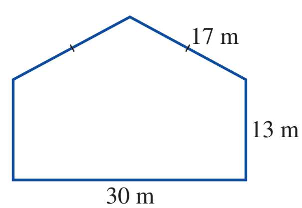


6 **WE6** Find the area of each of the composite figures drawn below.

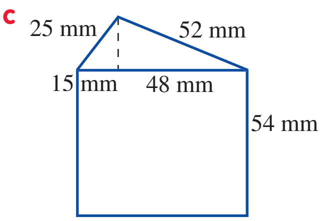
a



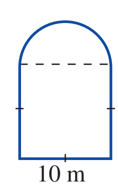
b



c

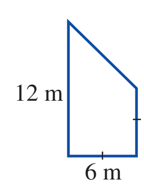


7 **MC** The area of the composite figure on the right is closest to:



A 139 m^2
B 257 m^2
C 314 m^2
D 414 m^2

8 **MC** The area of the figure drawn on the right is:

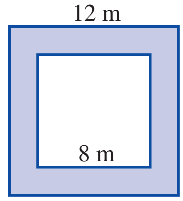


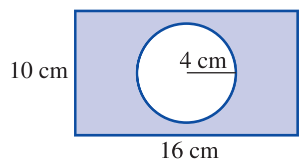
A 36 m^2
B 54 m^2
C 72 m^2
D 144 m^2

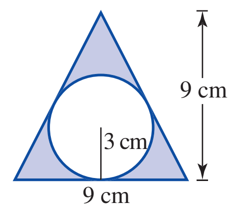
9 A block of land is in the shape of a square with an equilateral triangle on top. Each side of the block of land is 50 m.

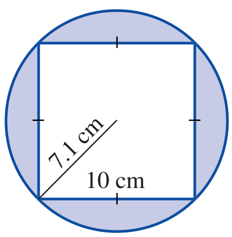
a Draw a diagram of the block of land.
b Find the perimeter of the block of land.
c Find the area of the block of land.

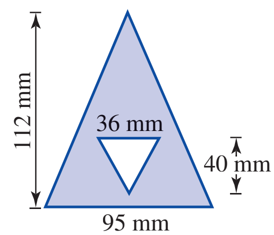
10 WE7 In each of the following, find the area of the shaded region. Where necessary, give your answer correct to 1 decimal place.

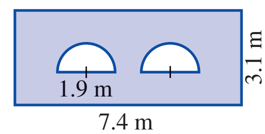
a


b


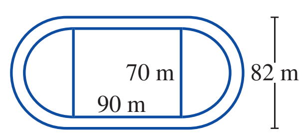
c


d


e


f


11 An athletics track consists of a rectangle with two semicircular ends. The dimensions are shown in the diagram below.



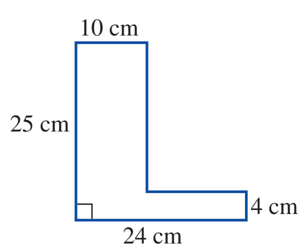
The track is to have a synthetic running surface laid. Calculate the area which is to be laid with the running surface, correct to the nearest square metre.

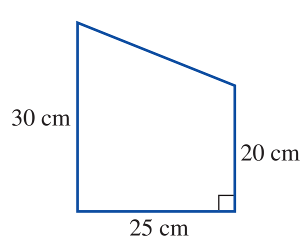
12 A garden is to have a concrete path laid around it. The garden is rectangular in shape and measures 40 m by 25 m. The path around it is to be 1 m wide.

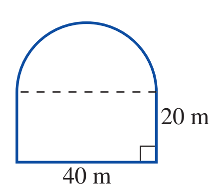
a Draw a diagram of the garden and the path.
b Calculate the area of the garden.
c Calculate the area of the concrete that needs to be laid.
d If the cost of laying concrete is \$17.50 per m^2, calculate the cost of laying the path.

Further development

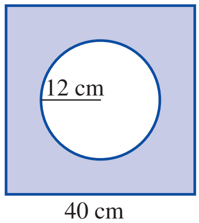
13 Find the area of each of the following figures.

a


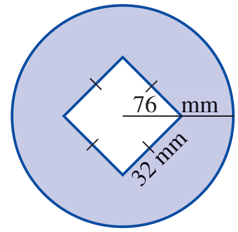
b


c


d



e



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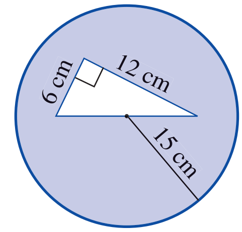
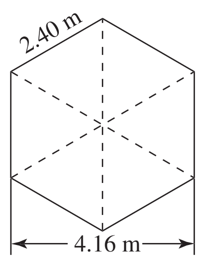


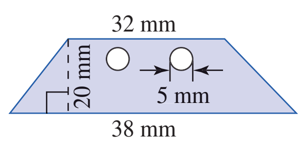
14 Convert the following areas to the units given in brackets.

a 20 000 mm^2 (cm^2) **b** 3 500 000 cm^2 (m^2) **c** 0.005 m^2 (cm^2)
d 0.035 m^2 (mm^2) **e** 13 400 m^2 (km^2) **f** 375 000 m^2 (hectares)
g 2 750 000 000 mm^2 (m^2) **h** 0.043 km^2 (m^2)

15 Find the area of the regular hexagon shown in the diagram below.

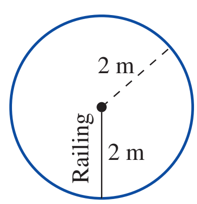


16 A cutting blade for a craft knife has the dimensions shown in the diagram. What is the area of steel in the blade (to the nearest mm^2)?



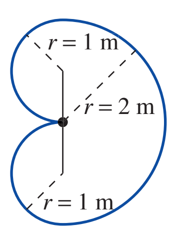
17 Emma left her horse tied to a railing in a paddock while she chatted to a friend.

a The horse is tied to one end of the railing as shown below.



Find the area that the horse has access to correct to 1 decimal place.

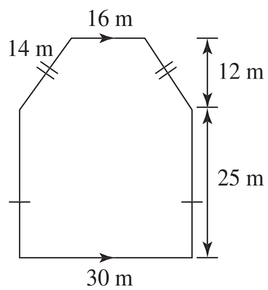
b The horse is now tied to the centre of the railing as shown below.



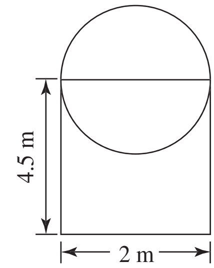
Find the area that the horse now has access to.

18 Find the areas of the following figures (to 1 decimal place).

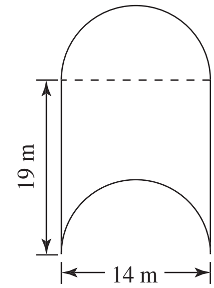
a



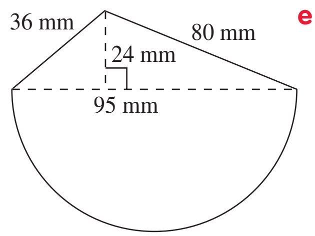
b



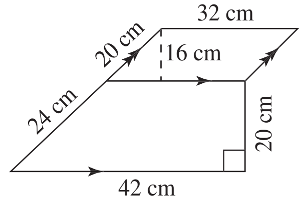
c



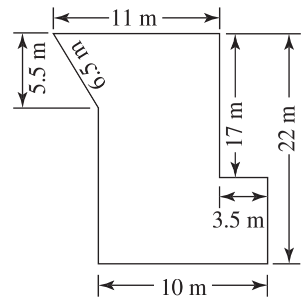
d



e

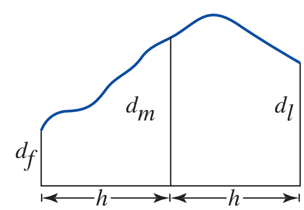


f



2C Simpson's rule

Simpson's rule is a method used to approximate the area of an irregular figure. Simpson's rule approximates an area by taking a straight boundary and dividing the area into two strips. The height of each strip (h) is measured. Three measurements are then taken perpendicular to the straight boundary, as shown in the figure on the right. The formula for Simpson's rule is:

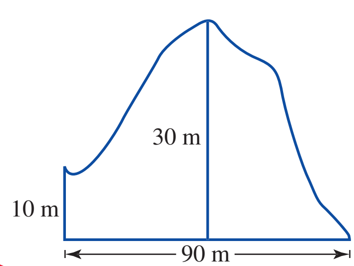


$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$

where h = distance between successive measurements
d_f = first measurement
d_m = middle measurement
d_l = last measurement.

WORKED EXAMPLE 8

Use Simpson's rule to approximate the area shown on the right.



THINK

1. Calculate h.
2. Write down the values of d_f, d_m and d_l.

WRITE

$h = 90 \div 2 = 45$

$d_f = 10,\ d_m = 30,\ d_l = 0$

3 Write the formula. $A \approx \frac{h}{3}(d_f + 4d_m + d_l)$

4 Substitute. $A \approx \frac{45}{3}(10 + 4 \times 30 + 0)$

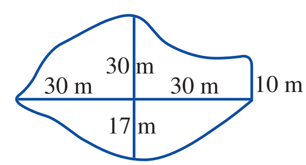
5 Calculate. $= 15 \times 130$
$\approx 1950\text{ m}^2$

Could Simpson's rule be used to estimate the areas of these irregular shapes from nature?

Simpson's rule can be used to approximate the area of an irregular shape without a straight edge. This is done by constructing a line as in the diagram in the worked example below and approximating the area of each section separately.

WORKED EXAMPLE 9

Use Simpson's rule to find an approximation for the area shown on the right.



THINK	WRITE
1 Write down the value of h.	$h = 30$
2 For the top area, write down the values of d_f, d_m and d_l.	$d_f = 0$, $d_m = 30$, $d_l = 10$
3 Write the formula.	$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$
4 Substitute.	$A \approx \frac{30}{3}(0 + 4 \times 30 + 10)$
5 Calculate the top area.	$\approx 10 \times 130$ $\approx 1300\text{ m}^2$
6 For the bottom area, write down the values of d_f, d_m and d_l.	$d_f = 0$, $d_m = 17$, $d_l = 0$

7 Write down the formula. $A \approx \frac{h}{3}(d_f + 4d_m + d_l)$

8 Substitute. $A \approx \frac{30}{3}(0 + 4 \times 17 + 0)$

9 Calculate the bottom area. $\approx 10 \times 68$

$\approx 680\ \text{m}^2$

10 Add the two areas together. $\text{Area} \approx 1300 + 680$

$\approx 1980\ \text{m}^2$

Simpson's rule approximates an area, it does not give an exact measurement. To obtain a better approximation, Simpson's rule can be applied several times to the area. This is done by splitting the area in half and applying Simpson's rule separately to each half.

WORKED EXAMPLE 10

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Tutorial
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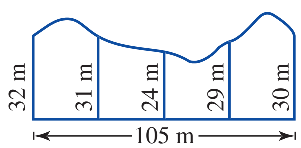
Worked example 10

Use two applications of Simpson's rule to approximate the area on the right.



THINK

WRITE

1 Calculate h by dividing 105 by 4. (We are using 4 sub-intervals.)

$h = 105 \div 4$

$= 26.25$

2 Apply Simpson's rule to the left half. Write the values of d_f, d_m and d_l.

$d_f = 32,\ d_m = 31,\ d_l = 24$

3 Write the formula.

$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$

4 Substitute.

$A \approx \frac{26.25}{3}(32 + 4 \times 31 + 24)$

5 Calculate the approximate area of the left half.

$\approx 8.75 \times 180$

$\approx 1575\ \text{m}^2$

6 Apply Simpson's rule to the right half. Write the values of d_f, d_m and d_l.

$d_f = 24,\ d_m = 29,\ d_l = 30$

7 Write the formula.

$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$

8 Substitute.

$A \approx \frac{26.25}{3}(24 + 4 \times 29 + 30)$

9 Calculate the approximate area of the right half.

$\approx 8.75 \times 170$

$\approx 1487.5\ \text{m}^2$

10 Add the areas together.

$\text{Area} \approx 1575 + 1487.5$

$\approx 3062.5\ \text{m}^2$

REMEMBER

1. Simpson's rule is a method of approximating irregular areas.
2. The Simpson's rule formula is $A \approx \frac{h}{3}(d_f + 4d_m + d_l)$, where h is the distance between successive measurements, d_f is the first measurement, d_m is the middle measurement and d_l is the last measurement.
3. A better approximation of an area can be found by using Simpson's rule several times.

EXERCISE 2C Simpson's rule

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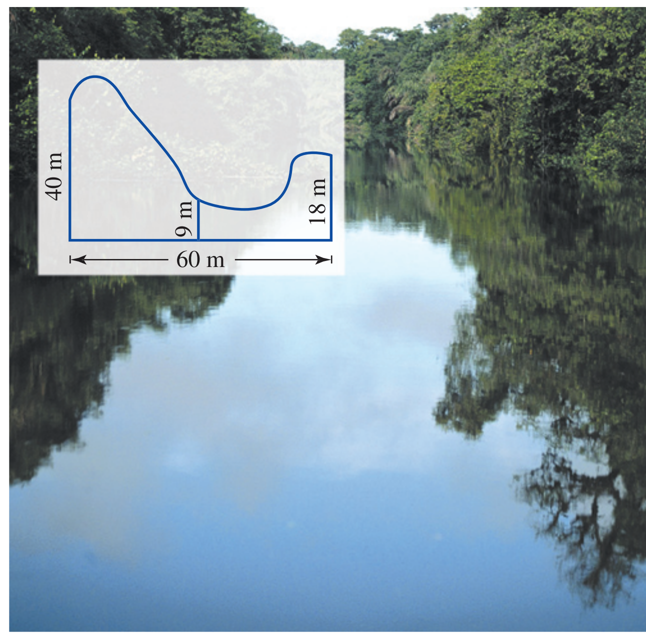
doc-1308
Substitution into formulas

1 WE8 The diagram on the right is of a part of a creek.

a State the value of h.

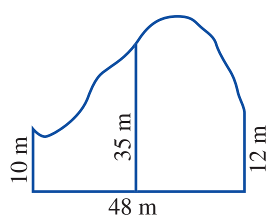
b State the value of d_f, d_m and d_l.

c Use Simpson's rule to approximate the area of this section of the creek.

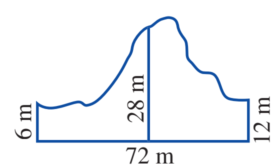


2 Use Simpson's rule to approximate each of the areas below.

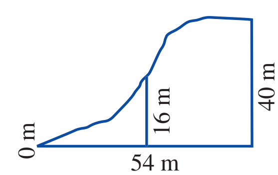
a



b



c

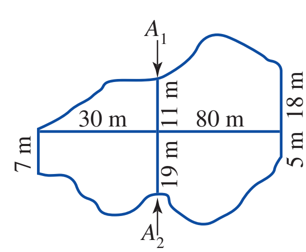


3 WE9 The irregular area on the right has been divided into two areas labelled A_1 (upper area) and A_2 (lower area).

a Use Simpson's rule to find an approximation for A_1.

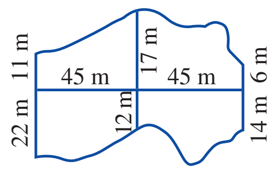
b Use Simpson's rule to find an approximation for A_2.

c What is the approximate total area of the figure?

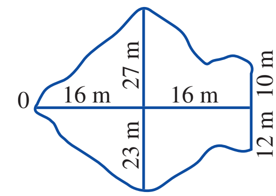


4 Use Simpson's rule to find an approximation for each of the areas below.

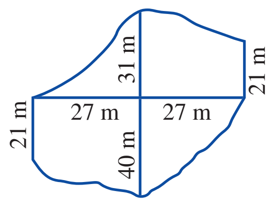
a



b



c



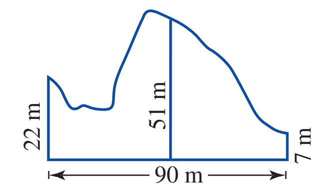
5 MC Consider the figure drawn on the right. Simpson's rule gives an approximate area of:

A $1200\ m^2$

B $2400\ m^2$

C $3495\ m^2$

D $6990\ m^2$



6 MC If we apply Simpson's rule twice, how many measurements from the traverse line need to be taken?

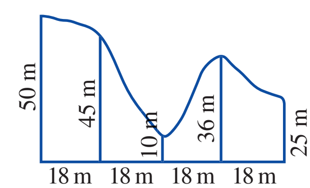
A 4

B 5

C 7

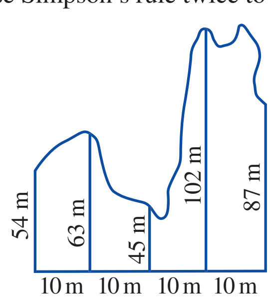
D 9

7 WE10 Use Simpson's rule twice to approximate the area on the right.

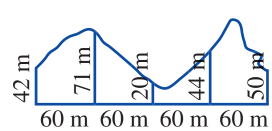


8 Use Simpson's rule twice to approximate each of the areas drawn below.

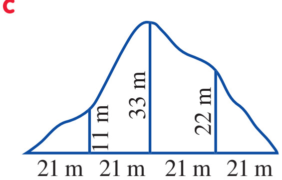
a



b



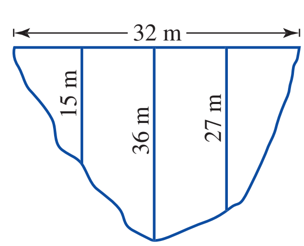
c



9 The figure on the right is of a cross-section of a waterway.

a Use Simpson's rule once to find an approximate area of this section of land.

b Use Simpson's rule twice to obtain a better approximation for the cross-section.



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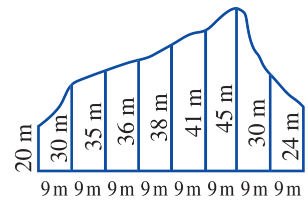
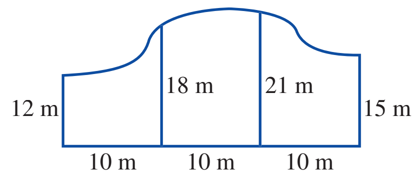
10 Apply Simpson's rule four times to approximate the area on the right.

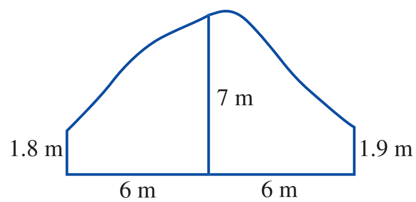


Further development

11 Explain why Simpson's rule can not be used to find the area of the figure below.

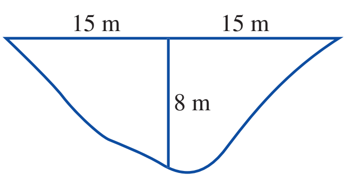


12 The figure below shows the entrance to a cave.



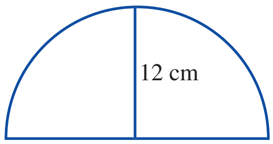
a Find the area of the entrance.
b The cave is 25 metres long and approximately has the same cross-section for its entire depth. Approximate the volume of the cave.

13 The figure below shows the cross section of a river.



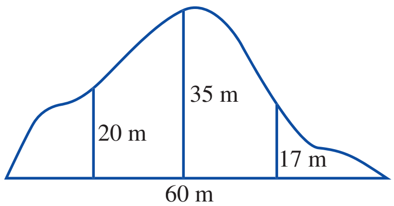
Find the area of the cross-section.

14 The figure below shows a semicircle of radius 12 cm.



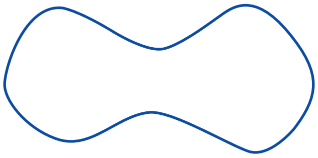
a Find the area of the semicircle correct to the nearest cm^2.
b Joe finds the area of the semicircle using Simpson's rule, taking $h = 12$ cm, the middle value as 12 cm, while the first and last values are both zero. What answer does Joe get?
c Find the percentage error in using Simpson's rule to find the area of this semicircle.

15 A botanist needs to estimate the number of trees in a certain headland area. It is known that there are approximately 32 trees in every 100 square metres. A diagram of the area is drawn below.



Complete the estimate of the number of trees to the nearest 10 trees.

16 The figure below is a top view of an in ground swimming pool.

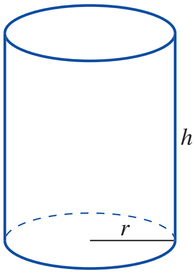


The pool is symmetrical and has a length of 16 metres. At its two widest points the width of the pool is 10 metres and at the narrow point in the middle it is 5 metres wide. The pool is 2.5 metres deep at all points.
Use Simpson's rule to estimate the volume of the pool.

2D Surface area of cylinders and spheres

From earlier work you should remember that surface area is the area of all surfaces of a 3-dimensional shape.

Consider a closed cylinder with a radius (r) and a perpendicular height (h). The surface of the cylinder consists of two circles and a rectangle.



$$\text{Area of top} = \pi r^2$$
$$\text{Area of bottom} = \pi r^2$$

The rectangular side of the cylinder will have a length equal to the circumference of the circle ($2\pi r$) and a width equal to the height (h) of the cylinder.

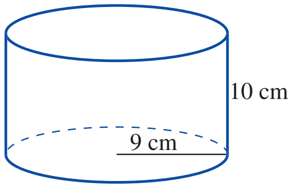
$$\text{Area of side} = 2\pi rh$$

The surface area of the closed cylinder can be calculated using the formula:

$$SA = 2\pi r^2 + 2\pi rh$$

WORKED EXAMPLE 11

Calculate the surface area of the closed cylinder drawn on the right. Give your answer correct to 1 decimal place.



THINK

1. Write the formula.
2. Substitute the values of r and h.
3. Calculate the surface area.

WRITE

$$SA = 2\pi r^2 + 2\pi rh$$
$$= 2 \times \pi \times 9^2 + 2 \times \pi \times 9 \times 10$$
$$= 1074.4\,\text{cm}^2$$

For cylinders, before calculating the surface area you need to consider whether the cylinder is open or closed. In the case of an open cylinder there is no top and so the formula needs to be written as:

$$SA = \pi r^2 + 2\pi rh$$

Note: On the formula sheet in the exam only the formula for the closed cylinder is provided. You will need to check the question and adapt the formula yourself if necessary.

WORKED EXAMPLE 12

Calculate the surface area of an open cylinder with a radius of 6.5 cm and a height of 10.8 cm. Give your answer correct to 1 decimal place.

THINK

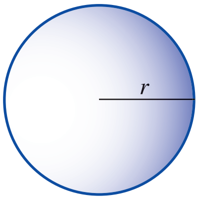
1. Write the formula.
2. Substitute the values of r and h.
3. Calculate the surface area.

WRITE

$$SA = \pi r^2 + 2\pi rh$$
$$= \pi \times (6.5)^2 + 2 \times \pi \times 6.5 \times 10.8$$
$$= 573.8\,\text{cm}^2$$

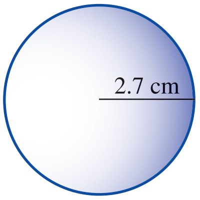
A sphere is a round 3-dimensional shape, and the only measurement given is the radius (r). The surface area of a sphere can be calculated using the formula:

$$SA = 4\pi r^2$$



WORKED EXAMPLE 13

Calculate the surface area of the sphere drawn on the right. Give the answer correct to 1 decimal place.



THINK	WRITE
1 Write the formula.	$SA = 4\pi r^2$
2 Substitute the value of r.	$= 4 \times \pi \times (2.7)^2$
3 Calculate the surface area.	$= 91.6\ \text{cm}^2$

REMEMBER

1. The surface area of a closed cylinder is found using the formula $SA = 2\pi r^2 + 2\pi rh$.
2. If the cylinder is an open cylinder, the surface area formula becomes $SA = \pi r^2 + 2\pi rh$.
3. The surface area of a sphere is found using the formula $SA = 4\pi r^2$.

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EXERCISE 2D Surface area of cylinders and spheres

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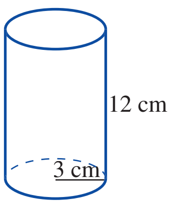
Circumference of a circle

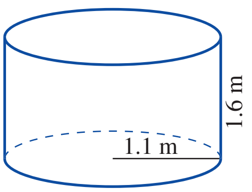
1 WE11 Calculate the surface area of a closed cylinder with a radius of 5 cm and a height of 11 cm. Give your answer correct to 1 decimal place.

2 Calculate the surface area of each of the closed cylinders drawn below. Give each answer correct to 1 decimal place.

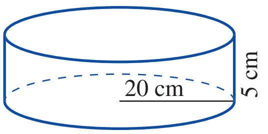
a



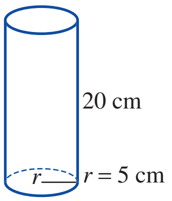
b



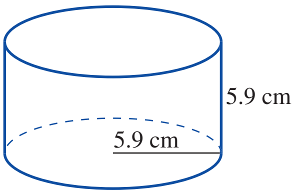
c



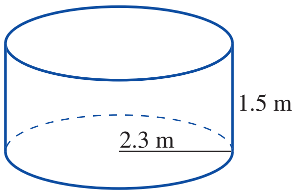
d



e



f

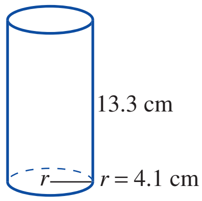


3 Calculate the surface area of a closed cylinder with a diameter of 3.4 m and a height of 1.8 m. Give your answer correct to 1 decimal place.

4 WE12 Calculate the surface area of an open cylinder with a radius of 4 cm and a height of 16 cm. Give your answer correct to the nearest whole number.

5 Calculate the surface area of each of the following open cylinders. Give each answer correct to 1 decimal place.

a



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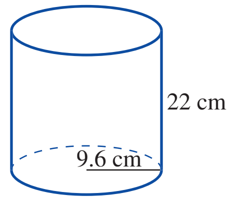
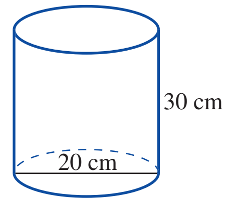


c



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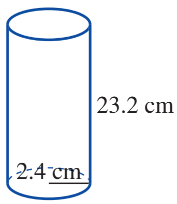

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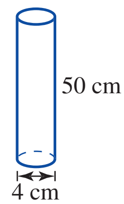

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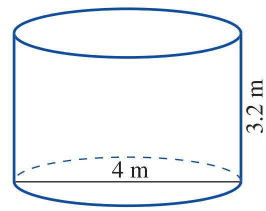


6 Find the outside surface area of a cylinder open at both ends with a radius of 5 cm and a height of 10 cm.

7 A can of fruit is made of stainless steel. The can has a radius of 3.5 cm and a height of 7 cm. A label is to be wrapped around the can.

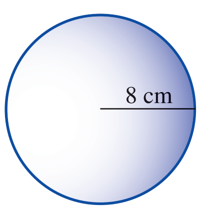
a Calculate the amount of steel needed to make the can (correct to the nearest whole number).

b Calculate the area of the label (correct to the nearest whole number).

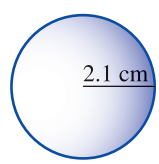
8 WE13 Calculate the surface area of a sphere with a radius of 3 cm. Give your answer correct to the nearest whole number.

9 Calculate the surface area of each of the spheres drawn below. Give each answer correct to 1 decimal place.

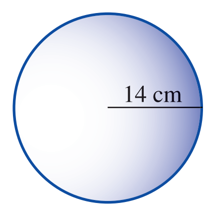
a



b



c



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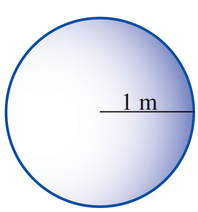
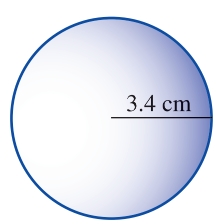


e



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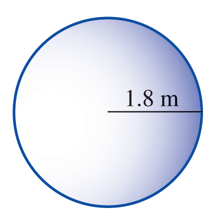


10 Calculate the surface area of a sphere with a diameter of 42 cm. Give your answer correct to the nearest whole number.

11 MC An open cylinder has a diameter of 12 cm and a height of 15 cm. Which of the following calculations gives the correct surface area of the cylinder?

A $\pi \times 6^2 + 2 \times \pi \times 6 \times 15$

B $2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 15$

C $\pi \times 12^2 + 2 \times \pi \times 12 \times 15$

D $2 \times \pi \times 12^2 + 2 \times \pi \times 12 \times 15$

12 MC Which of the following figures has the greatest surface area?

A A closed cylinder with a radius of 5 cm and a height of 10 cm
B An open cylinder with a radius of 6 cm and a height of 10 cm
C A cylinder open at both ends with a radius of 7 cm and a height of 10 cm
D A sphere with a radius of 6 cm

13 An open cylinder has a diameter and height of 12 cm.

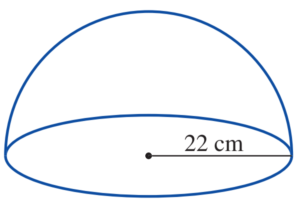
a Calculate the surface area of the cylinder (correct to the nearest whole number).
b A sphere sits exactly inside this cylinder. Calculate the surface area of this sphere (correct to the nearest whole number).

Further development

14 A cylindrical can is to contain three tennis balls each having a diameter of 6 cm.

a Calculate the surface area of each ball.
b The three balls fit exactly inside the can. State the radius and height of the can.
c The can is open and made of stainless steel, except the top which will be plastic. Calculate the area of the plastic lid (correct to the nearest whole number).
d Calculate the amount of stainless steel in the can (correct to the nearest whole number).
e Calculate the area of a paper label that is to be wrapped around the can (correct to the nearest whole number).

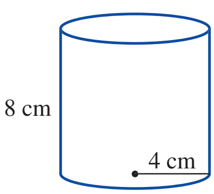
15 Calculate the surface area of the hemisphere drawn below given that it is open at the base.



Give your answer correct to the nearest hundred cm^2.

16 Calculate the outside surface area of the hemisphere in question **15** if it is closed at the base.

17 Find the surface area of the largest sphere that can be placed inside the cylinder right.



18 A tennis ball has a diameter of 7 cm. Calculate the surface area of a tennis ball canister that is to hold four tennis balls.

19 Soccer balls have a diameter of 30 cm.

a The soccer ball is to be placed in the smallest possible cubic box. Calculate the surface area of this box.
b Calculate the percentage material saved by placing the ball in a spherical box of diameter 30 cm. Give your answer to the nearest whole number.

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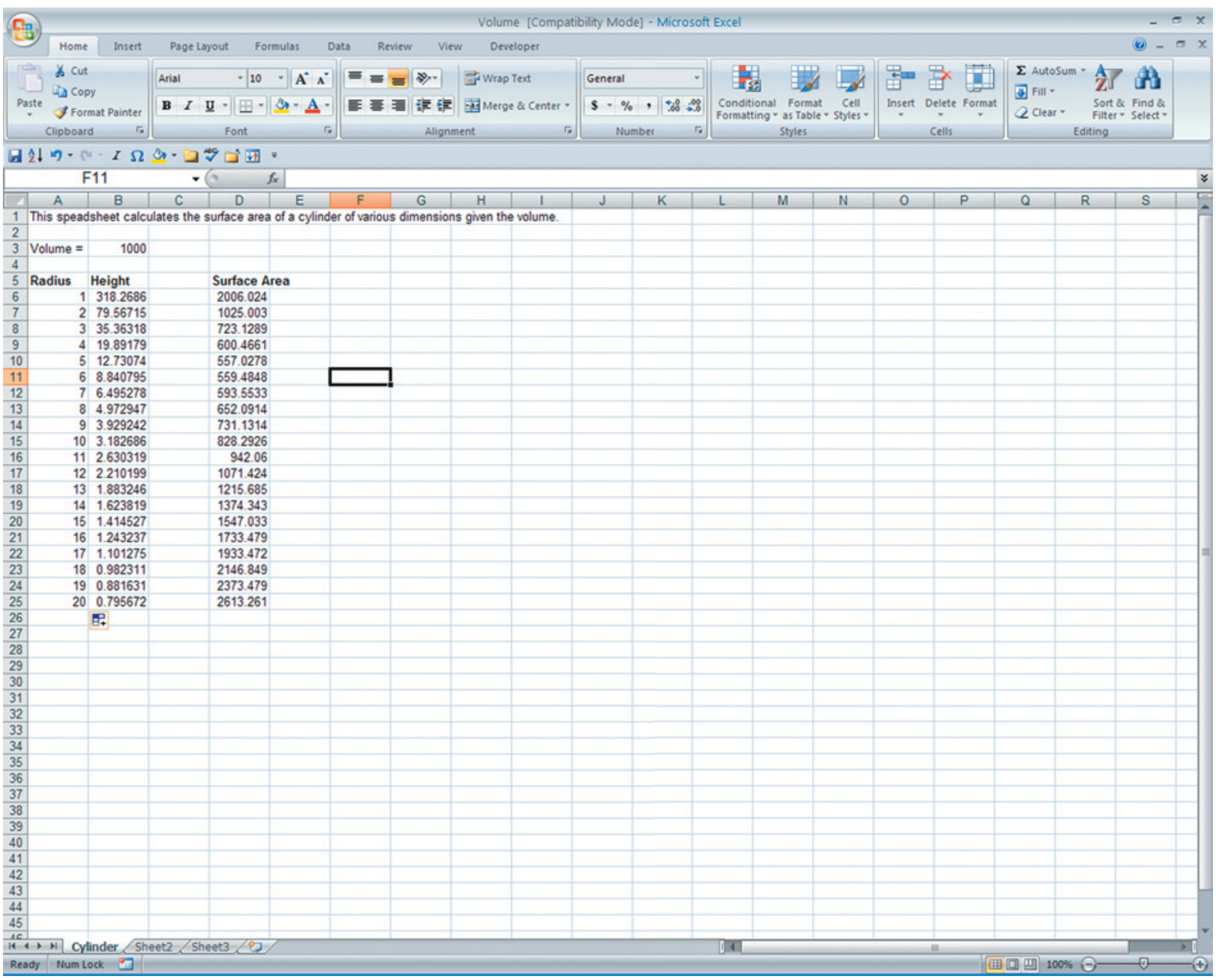
Volume

Computer Application 1: Minimising surface area

Access the spreadsheet *Volume* from the *Maths Quest General Mathematics HSC Course* eBookPLUS.

A cylindrical drink container is to have a capacity of 1 litre (volume = 1000 cm^3). We are going to calculate the most cost efficient dimensions to make the container. To do this, we want to make the container with as little material as possible, in other words we want to minimise the surface area of the cylinder. The spreadsheet should look as shown below.

1. In cell **B3** enter the volume of the cylinder, 1000.
2. In cell **A6** enter a radius of 1. In cell **A7** enter a radius of 2 and so on up to a radius of 20.
3. The formula that has been entered in cell **B6** will give the height of the cylinder corresponding to the radius for the given volume.
4. The surface area of each possible cylinder is in column D. Use the charting function on the spreadsheet to graph the surface area against the radius.
5. What are the most cost-efficient dimensions of the drink container?



This speadsheet calculates the surface area of a cylinder of various dimensions given the volume.

Volume =	1000	
Radius	Height	Surface Area
1	318.2686	2006.024
2	79.56715	1025.003
3	35.36318	723.1289
4	19.89179	600.4661
5	12.73074	557.0278
6	8.840795	559.4848
7	6.495278	593.5533
8	4.972947	652.0914
9	3.929242	731.1314
10	3.182686	828.2926
11	2.630319	942.06
12	2.210199	1071.424
13	1.883246	1215.685
14	1.623819	1374.343
15	1.414527	1547.033
16	1.243237	1733.479
17	1.101275	1933.472
18	0.982311	2146.849
19	0.881631	2373.479
20	0.795672	2613.261

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Challenge exercise

Use one of the other worksheets to find the most efficient dimensions to make a rectangular prism of volume 1000 cm^3 and a cone of volume 200 cm^3.

2E Volume of composite solids

Many solid shapes are composed of two or more regular solids. To calculate the volume of such a figure, we need to determine the best method for each particular part. Many irregular shapes may still be prisms.

A prism is a shape in which every cross-section taken parallel to the base shape is equal to that base shape.

The formula for the volume of a prism is:

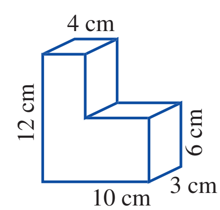
$$V = Ah$$

where A is the area of the base shape and h is the height.

Remember that the base of the prism is not necessarily the bottom. The base is the shape that is constant throughout the prism and will usually be drawn as the front of the prism. This means that the height will be drawn perpendicular to the base. To calculate the volume of any prism, we first calculate the area of the base and then multiply by the height.

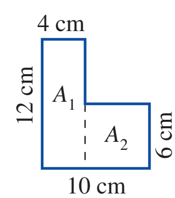
WORKED EXAMPLE 14

Find the volume of the figure drawn on the right.



THINK | WRITE

1. Divide the front face into two rectangles.



2. Calculate the area of each.

$A_1 = 4 \times 12$
$= 48\text{ cm}^2$

$A_2 = 6 \times 6$
$= 36\text{ cm}^2$

3. Add the areas together to find the value of A.

$A = 48 + 36$
$= 84\text{ cm}^2$

4. Write the formula.

$V = A \times h$

5. Substitute $A = 84$ and $h = 3$.

$= 84 \times 3$

6. Calculate.

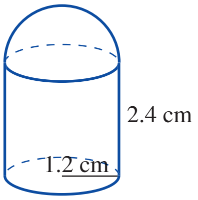
$= 252\text{ cm}^3$

If the shape is not a prism, you may need to divide it into two or more regular 3-dimensional shapes. You could then calculate the volume by finding the volume of each shape separately. You will need to use important volume formulas that appear on the formula sheet:

Cone: $V = \frac{1}{3}\pi r^2 h$ Cylinder: $V = \pi r^2 h$ Pyramid: $V = \frac{1}{3}Ah$ Sphere: $V = \frac{4}{3}\pi r^3$

WORKED EXAMPLE 15

Calculate the volume of the figure drawn on the right, correct to 2 decimal places.



THINK / **WRITE**

1. The shape is a cylinder with a hemisphere on top.
2. Write down the formula for the volume of a cylinder. — $V = \pi r^2 h$
3. Substitute $r = 1.2$ and $h = 2.4$. — $= \pi \times (1.2)^2 \times 2.4$
4. Calculate the volume of the cylinder. — $= 10.857\ \text{cm}^3$
5. Write down the formula for the volume of a hemisphere. (This is the formula for the volume of a sphere divided by 2.) — $V = \frac{4}{3}\pi r^3 \div 2$
6. Substitute $r = 1.2$. — $= \frac{4}{3} \times \pi \times (1.2)^3 \div 2$
7. Calculate the volume of the hemisphere. — $= 3.619\ \text{cm}^3$
8. Add the two volumes together. — Volume $= 10.857 + 3.619$
   $= 14.48\ \text{cm}^3$

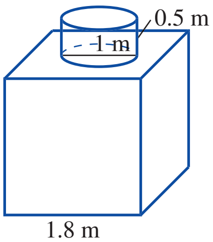
In many cases a volume question may be presented in the form of a practical problem.

WORKED EXAMPLE 16

A water storage tank is in the shape of a cube of side length 1.8 m, surmounted by a cylinder of diameter 1 m with a height of 0.5 m. Calculate the capacity of the tank, correct to the nearest 100 litres.

THINK / **WRITE**

1. Draw a diagram of the water tank.



2. Calculate the volume of the cube using the formula $V = s^3$. — $V = s^3$
   $= 1.8^3$
   $= 5.832\ \text{m}^3$
3. Calculate the volume of the cylinder using the formula $V = \pi r^2 h$. — $V = \pi r^2 h$
   $= \pi \times 0.5^2 \times 0.5$
   $= 0.393\ \text{m}^3$

4	Add the volumes together.	Volume $= 5.832 + 0.393$ $= 6.225\text{ m}^3$
5	Calculate the capacity of the tank using $1\text{ m}^3 = 1000\text{ L}$.	Capacity $= 6.225 \times 1000$ $= 6225\text{ L}$
6	Give an answer in words.	The capacity of the tank is approximately 6200 litres.

REMEMBER

1. To find the volume of any prism, use the formula $V = A \times h$, where A is the area of the base and h is the height.
2. Important volume formulas:

$$\text{Cone: } V = \tfrac{1}{3}\pi r^2 h \qquad \text{Cylinder: } V = \pi r^2 h$$
$$\text{Pyramid: } V = \tfrac{1}{3}Ah \qquad \text{Sphere: } V = \tfrac{4}{3}\pi r^3$$

   where r = radius, h = perpendicular height, A = area of base.
3. For other shapes, calculate the volume of each part of the shape separately, then add together each part at the end.
4. Remember to begin a worded or problem question with a diagram and finish with a word answer.

EXERCISE 2E Volume of composite solids

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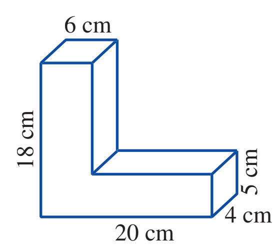
Volume of cubes and rectangular prisms

1 WE14 Look at the figure drawn on the right.

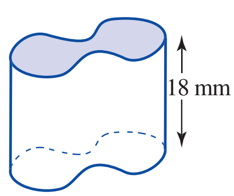
a Find the area of the front face.

b Use the formula $V = A \times h$ to calculate the volume of the prism.



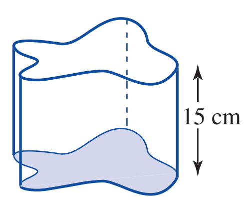
2 Find the volume of the following prisms (to 2 decimal place).

a



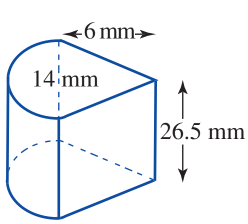
[Base area: 35 mm²]

b

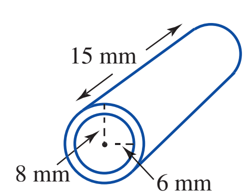


[Base area: 28 cm²]

c

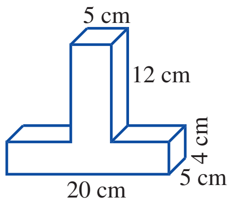


d

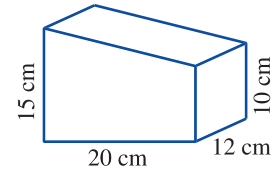


3 Calculate the volume of each of the figures drawn below.

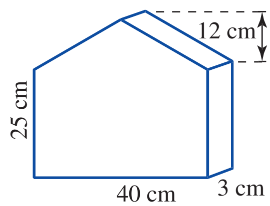
a



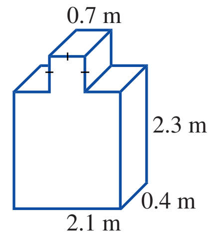
b



c



d



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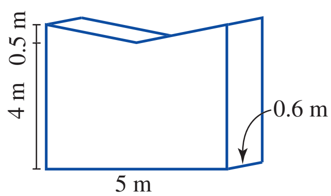

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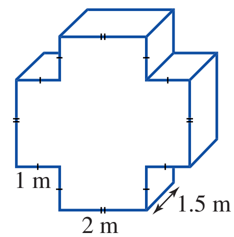

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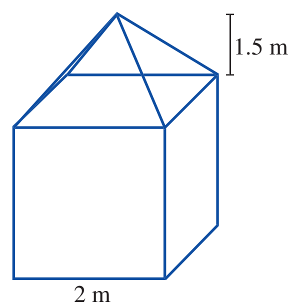
Volume of triangular prisms

4 **WE15** Consider the figure on the right. The shape consists of a cube with a square pyramid on top.

a What is the volume of the cube?

b What is the volume of the square pyramid?

c What is the total volume of this figure?

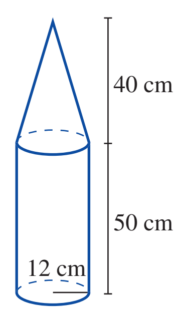


5 The figure on the right is a cylinder with a cone mounted on top.

a Calculate the volume of the cylinder, correct to the nearest cm^3.

b Calculate the volume of the cone, correct to the nearest cm^3.

c What is the total volume of the figure?



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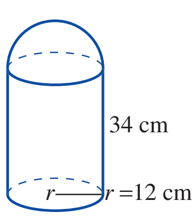
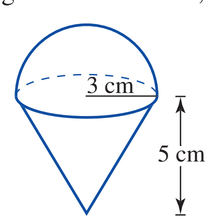
Volume of cylinders

6 Calculate the volume of each of the figures drawn below, correct to 1 decimal place.

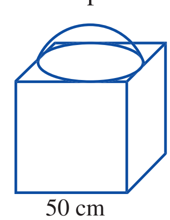
a



b



c



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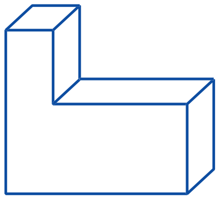
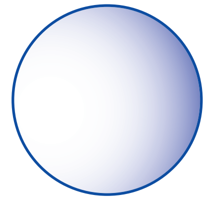
doc-1315
Volume of a sphere

7 MC Which of the figures drawn below is *not* a prism?

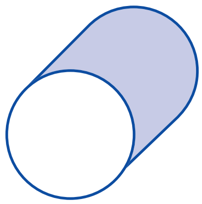
A



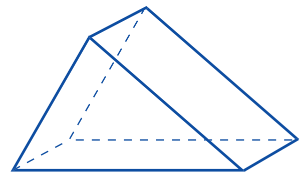
B



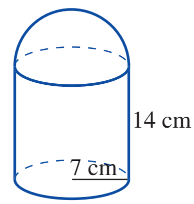
C



D



8 MC The volume of the figure drawn on the right is closest to:



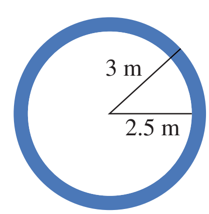
A 718 cm^3
B 1437 cm^3
C 2155 cm^3
D 2873 cm^3

9 A fish tank is in the shape of a rectangular prism. The base measures 45 cm by 25 cm. The tank is filled to a depth of 15 cm.

a Calculate the volume of water in the tank in cm^3.
b Given that 1 cm^3 = 1 mL calculate, in litres, the amount of water in the tank.

10 WE16 A hemispherical wine glass of radius 2.5 cm is joined to a cylinder of radius 1 cm and height 5 cm. The glass then rests on a solid base.
a Draw a diagram of the wine glass.
b Calculate the capacity of the glass, to the nearest 10 mL.
c How many glasses of wine can be poured from a 1 litre bottle?

11 The figure on the right is the cross-section of a concrete pipe used as a sewage outlet.



a Calculate the area of a cross-section of the pipe, correct to 2 decimal places.
b Calculate the amount of concrete needed to make a 10 m length of this pipe.

12 A commemorative cricket ball has a diameter of 7 cm. It is to be preserved in a cubic case that will allow 5 mm on each side of the ball.
a What will the side length of the cubic case be?
b Calculate the amount of empty space inside the case, to the nearest whole number.
c Calculate the percentage of space inside the case occupied by the ball, to the nearest whole number.

Further development

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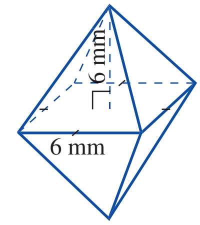
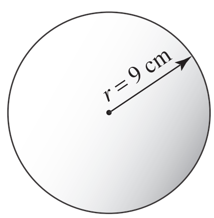
doc-1316
Volume of a pyramid

13 A diamond is cut into the shape of two square-based pyramids as shown on the right. Each mm^3 of the diamond has a mass of 0.04 g. Calculate the mass of the diamond.

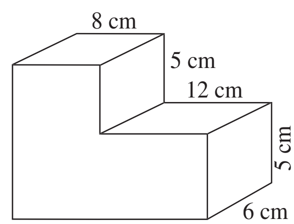


14 Find the volume of these objects (to the nearest whole unit).

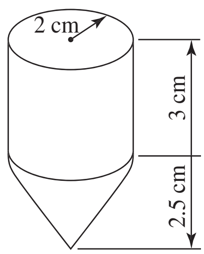
a



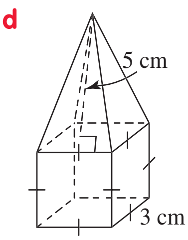
b



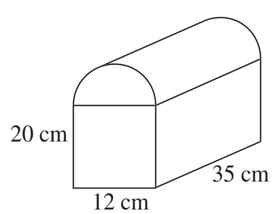
c



d



e



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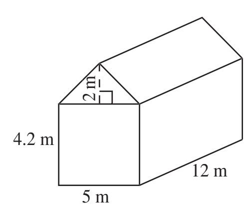
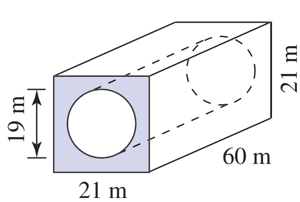


g



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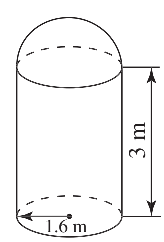
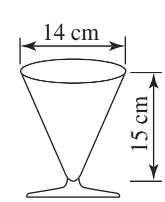


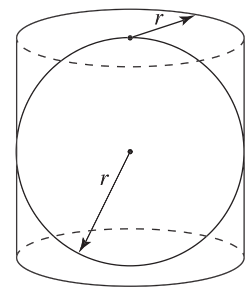
15 The medicine cup on the right has the shape of a cone with a diameter of 4 cm and a height of 5 cm (not including the cup's base). Find the volume of the cone to the nearest millilitre, where 1 cm^3 = 1 mL.



16 Tennis balls have a diameter of 6.5 cm and are packaged in a cylinder that can hold four tennis balls. Assuming the balls just fit inside a cylinder, find:

a the height of the cylindrical can
b the volume of the can (to 1 decimal place)
c the volume of the four tennis balls (to 1 decimal place)
d the volume of the can occupied by air
e the fraction of the can's volume occupied by the balls.

17 **MC** The ratio of the volume of a sphere to that of a cylinder of similar dimensions, as shown in the diagram, is *best* expressed as:



A $\frac{4}{3}$ **B** $\frac{2}{3}$

C $\frac{3}{4}$ **D** $\frac{3}{2}$

18 A model aeroplane is controlled by a tethered string of 10 metres length. The operator stands in the middle of an oval. (Give all answers to the nearest whole unit.)

a What is the maximum area of the oval occupied by the plane in flight?
b If the plane can be manoeuvred in a hemispherical zone, find:
  i the surface area of the airspace that the plane can occupy
  ii the volume of airspace that is needed by the operator for controlling the plane.
c Repeat part **b** with a new control string length of 15 metres.

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Maximising volume

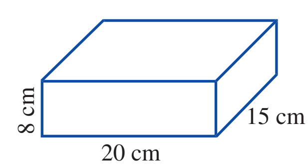
2F Error in measurement

As we saw in the preliminary course, all measurements are approximations. The degree of accuracy in any measurement is restricted by the accuracy of the measuring device and the degree of practicality.

We have previously seen that the maximum error in any measurement is half of the smallest unit of measurement. This error is compounded when further calculations such as surface area or volume are made.

WORKED EXAMPLE 17

In the rectangular prism on the right, the length, breadth and height have been measured, correct to the nearest centimetre.
a Calculate the volume of the rectangular prism.
b Calculate the greatest possible error in the volume.



THINK	WRITE
a Calculate the volume of the rectangular prism.	**a** $V = l \times w \times h$ $= 20 \times 15 \times 8$ $= 2400\text{ cm}^3$
b **1** Write the smallest possible dimensions of the prism.	**b** Smallest possible dimensions: $l = 19.5,\ w = 14.5,\ h = 7.5$
2 Calculate the smallest possible volume.	$V = l \times w \times h$ $= 19.5 \times 14.5 \times 7.5$ $= 2120.625\text{ cm}^3$

3. Write the largest possible dimensions of the prism.

Largest possible dimensions:
$l = 20.5$, $w = 15.5$, $h = 8.5$

4. Calculate the largest possible volume.

$V = l \times w \times h$
$= 20.5 \times 15.5 \times 8.5$
$= 2700.875\ \text{cm}^3$

5. Calculate the maximum error.

Maximum error $= 2700.875 - 2400$
$= 300.875\ \text{cm}^3$

As can be seen in the above example, a possible error of 0.5 cm in the linear measurement compounds to an error of $300.875\ \text{cm}^3$ in the volume measurement.

Errors in measurement will compound errors in all further calculations.

WORKED EXAMPLE 18

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Worked example 18

A swimming pool is built in the shape of a rectangular prism with a length of 10.2 m, a width of 7.5 m and a depth of 1.5 m. The floor and the sides of the pool need to be cemented.

a Calculate the area that is to be cemented.

b The concreter incorrectly measured the length of the pool as 9.4 m. Calculate the error in the area calculation.

c Calculate the percentage error (correct to 1 decimal place) in the area calculation.

THINK	WRITE
a 1. Calculate the area of the pool floor.	**a** Area of floor $= 10.2 \times 7.5$ $= 76.5\ \text{m}^2$
2. Calculate the area of the ends.	Area of ends $= 7.5 \times 1.5$ $= 11.25\ \text{m}^2$
3. Calculate the area of the sides.	Area of sides $= 10.2 \times 1.5$ $= 15.3\ \text{m}^2$
4. Calculate the total area to be cemented.	Total area $= 76.5 + 2 \times 11.25 + 2 \times 15.3$ $= 129.6\ \text{m}^2$
b 1. Use the incorrect measurement to repeat all the above calculations.	**b** Area of floor $= 9.4 \times 7.5$ $= 70.5\ \text{m}^2$ Area of ends $= 7.5 \times 1.5$ $= 11.25\ \text{m}^2$ Area of sides $= 9.4 \times 1.5$ $= 14.1\ \text{m}^2$ Total area $= 70.5 + 2 \times 11.25 + 2 \times 14.1$ $= 121.2\ \text{m}^2$

2 Find the difference between the two answers.

$$\text{Error} = 129.6 - 121.2$$
$$= 8.4\ \text{m}^2$$

c Write the error as a percentage of the correct answer.

c $$\text{Percentage error} = \frac{8.4}{129.6} \times 100\%$$
$$= 6.5\%$$

REMEMBER

1. All measurements are approximations. The accuracy of any measurement is limited by the instrument used and the most practical degree of accuracy.
2. The maximum error in any linear measurement is half the smallest unit used.
3. Any error made in linear measurement will compound when used in further calculations such as those for surface area or volume.

EXERCISE 2F Error in measurement

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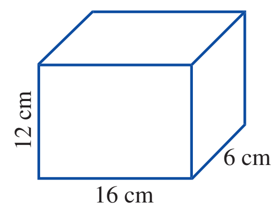
Error in linear measurement

1 WE17 In the figure on the right each measurement has been taken to the nearest centimetre.

a Calculate the volume of the figure.

b Calculate the maximum error in the volume calculation.



2 The radius of a circle is measured as 7.6 cm, correct to 1 decimal place.

a What is the maximum possible error in the measurement of the radius?

b Calculate the area of the circle. Give your answer correct to 1 decimal place.

c Calculate the maximum possible error in the area of the circle.

d Calculate the maximum possible error in the area of the circle as a percentage of the area.

3 A cube has a side length of 16 mm, correct to the nearest millimetre.

a Calculate the volume of the cube.

b Calculate the smallest possible volume of the cube.

c Calculate the largest possible volume of the cube.

d Calculate the maximum possible percentage error in the volume of the cube.

e Calculate the surface area of the cube.

f Calculate the smallest possible surface area of the cube.

g Calculate the largest possible surface area of the cube.

h Calculate the maximum possible percentage error in the surface area of the cube.

4 A cylinder has a radius of 4 cm and a height of 6 cm with each measurement being taken correct to the nearest centimetre.

a Calculate the volume of the cylinder (correct to the nearest whole number).

b Calculate the smallest possible volume of the cylinder (correct to the nearest whole number).

c Calculate the largest possible volume of the cylinder (correct to the nearest whole number).

d Calculate the greatest possible percentage error in the volume of the cylinder.

5 For the cylinder in question 4, calculate the greatest possible percentage error in the surface area of the cylinder.

6 The radius of a sphere is 1.4 m with the measurement taken correct to 1 decimal place.
  a Calculate the volume of the sphere, correct to 1 decimal place.
  b Calculate the maximum possible error in the volume of the sphere.
  c Calculate the maximum percentage error in the volume.
  d Calculate the surface area of the sphere, correct to 1 decimal place.
  e Calculate the maximum possible error in the surface area of the sphere.
  f Calculate the maximum percentage error in the surface area.

7 WE18 An open cylindrical water tank has a radius of 45 cm and a height of 60 cm.
  a Calculate the capacity of the tank, in litres (correct to the nearest whole number).
  b If the tank's radius is given as 50 cm, correct to the nearest 10 cm, calculate the error in the capacity of the tank.
  c Calculate the percentage error in the capacity of the tank.

8 A rectangular prism has dimensions 56 cm × 41 cm × 17 cm.
  a Calculate the volume of the prism.
  b Calculate the surface area of the prism.
  c If the dimensions are given to the nearest 10 cm, what will the dimensions of the prism be given as?
  d Calculate the percentage error in the volume when the dimensions are given to the nearest 10 cm.
  e Calculate the percentage error in the surface area when the dimensions are given to the nearest 10 cm.

9 The four walls of a room are to be painted. The length of the room is 4.1 m and the width is 3.6 m. Each wall is 1.8 m high.
  a Calculate the area to be painted.
  b One litre of paint will paint an area of 2 m^2. Each wall will need two coats of paint. Calculate the number of litres of paint required to complete this job.
  c Karla incorrectly measures the length of the room to be 3.9 m. If Karla does all her calculations using this incorrect measurement, how many litres will she be short of paint at the end of the job?

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10 The dimensions of a rectangular house are 16.6 m by 9.8 m.
  a Simon takes the dimensions of the house to the nearest metre for all his calculations. What dimensions does Simon use?
  b Simon plans to floor the house in slate tiles. What is the area that needs to be tiled?
  c The tiles cost \$27.50/m^2 and Simon buys an extra 10% to allow for cutting and breakage. Calculate the cost of the tiles.
  d How much extra has Simon spent than would have been necessary had he used the original measurements of the house?

Further development

11 The dimensions of a rectangular courtyard are 20 metres by 12 metres, correct to the nearest metre. The area is to be paved with pavers that are squares of side length 50 cm.

a Calculate the number of pavers that will be needed to ensure that the entire courtyard is paved, allowing for possible measurement error in the courtyard measurements.

b If this number of pavers are ordered what would be the maximum number of pavers that could be left over at the end of the job?

12 The area of a square is measured as being 4900 m^2, correct to the nearest 100 m^2. Find:

a the side length of the square

b the maximum possible side length (correct to 1 decimal place)

c the minimum possible side length (correct to 1 decimal place)

d the maximum percentage error in the side length (correct to 2 decimal places).

13 The volume of a sphere is found to be 23 000 cm^3, correct to the nearest 1000 cm^3. Find the maximum percentage error in:

a the volume of the sphere

b the radius of the sphere.

Give your answers correct to 2 decimal places.

14 A cylinder has given radius of 10 cm and a height of 30 cm, correct to the nearest centimetre.

a Find the volume of the cylinder, correct to the nearest cm^3.

b A liquid is to be poured into the cylinder. The liquid can expand by as much as 10% in hot weather. Allowing for possible error in measurement, what is the maximum amount of the liquid that can be poured into the cylinder such that none will spill in the event of expansion? Give your answer correct to the nearest 100 mL.

15 a Find the area of a sector of a circle of radius 15 cm and subtending a 74° angle at the centre.

b Find the length of the arc formed.

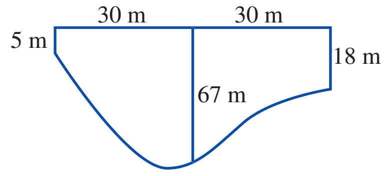
c Find the maximum percentage error in

i the area of the sector

ii the arc length

given that the radius was measured to the nearest centimetre and the angle was measured to the nearest degree.

16 The figure below is of a field. The area is to be approximated using Simpson's rule.



a Estimate the area.

b Given that each measurement is taken correct to the nearest metre, find the smallest possible area of the field.

c Find the largest possible area of the field.

d Find the maximum percentage error.

SUMMARY

Area of parts of the circle

- The area of a circle can be calculated using the formula $A = \pi r^2$.
- The area of a sector is found by multiplying the area of the full circle by the fraction of the circle occupied by the sector. This is calculated by looking at the angle that the sector makes with the centre.
- An annulus is the area between two circles. The area is calculated by subtracting the area of the smaller circle from the area of the larger circle or by using the formula $A = \pi(R^2 - r^2)$, where R is the radius of the large circle and r is the radius of the small circle.
- The area of an ellipse is calculated using the formula $A = \pi ab$, where a is the length of the semi-major axis and b is the length of the semi-minor axis.

Area of composite shapes

- The area of a composite figure is calculated by dividing the figure into two or more regular figures.
- When calculating the area of a composite figure, some side lengths will need to be calculated using Pythagoras' theorem.

Simpson's rule

- Simpson's rule is used to find an approximation for an irregular area.
- The formula for Simpson's rule is $A \approx \frac{h}{3}(d_f + 4d_m + d_l)$.
- To obtain a better approximation for an area, Simpson's rule can be applied twice. This is done by dividing the area in half and applying Simpson's rule separately to each half.

Surface area of cylinders and spheres

- The surface area of a closed cylinder is found by using the formula $SA = 2\pi r^2 + 2\pi rh$.
- If the cylinder is an open cylinder, the surface area is found using $SA = \pi r^2 + 2\pi rh$.
- The surface area of a sphere is calculated using the formula $SA = 4\pi r^2$.

Volume of composite solids

- The volume of solid prisms is calculated using the formula $V = A \times h$.
- The volume of a cone is found using the formula $V = \frac{1}{3}\pi r^2 h$.
- The volume of a cylinder is found using the formula $V = \pi r^2 h$.
- The volume of a sphere is found using the formula $V = \frac{4}{3}\pi r^3$.
- The volume of a pyramid is found using the formula $V = \frac{1}{3}Ah$.
- Other solids have their volume calculated by dividing the solid into regular solid shapes.

Error in measurement

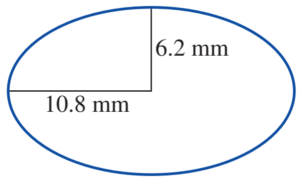
- All measurements are approximations. The maximum error in any measurement is half the smallest unit used.
- Any error in a measurement will compound when further calculations using the measurement need to be made.

CHAPTER REVIEW

MULTIPLE CHOICE

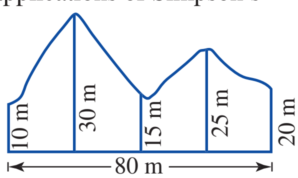
1 Which of the following calculations will correctly give the area of the ellipse drawn below?

A $\pi \times 6.2^2$
B $\pi \times 8.5^2$
C $\pi \times 10.8^2$
D $\pi \times 10.8 \times 6.2$



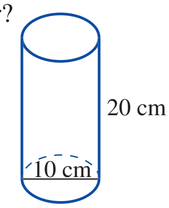
2 The field drawn below is to have its area approximated by two applications of Simpson's rule. The value of h is:

A 16
B 20
C 40
D 80



3 The figure drawn below is an open cylinder. Which of the calculations below will correctly give the surface area of the cylinder?

A $\pi \times 5^2 + 2 \times \pi \times 5 \times 20$
B $2 \times \pi \times 5^2 + 2 \times \pi \times 5 \times 20$
C $\pi \times 10^2 + 2 \times \pi \times 10 \times 20$
D $2 \times \pi \times 10^2 + 2 \times \pi \times 10 \times 20$



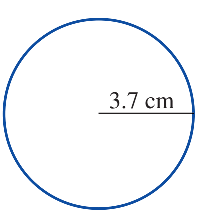
4 A closed cylinder is measured as having a radius of 1.2 m and a height of 1.4 m. The maximum error in the calculation of the surface area is:

A $1.2\ \text{m}^2$ **B** $1.5\ \text{m}^2$
C $1.6\ \text{m}^2$ **D** $19.6\ \text{m}^2$

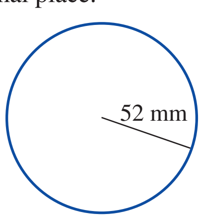
SHORT ANSWER

1 Calculate the area of each of the circles below. Give each answer correct to 1 decimal place.

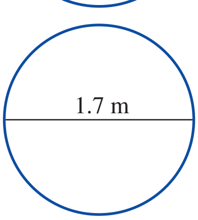
a



b

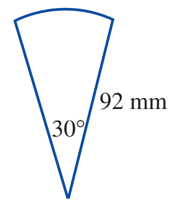


c

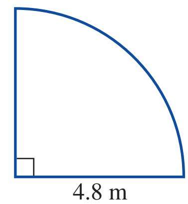


2 Calculate the area of each of the figures below. Give each answer correct to 1 decimal place.

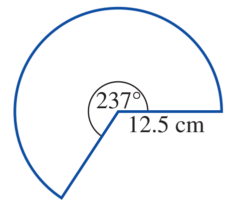
a



b

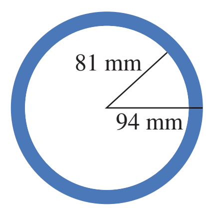


c

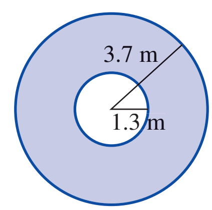


3 Calculate the area of each of the annuluses below. Give each answer correct to 1 decimal place.

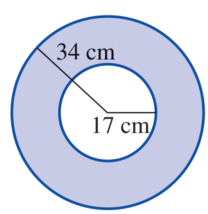
a



b

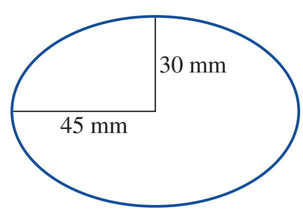


c

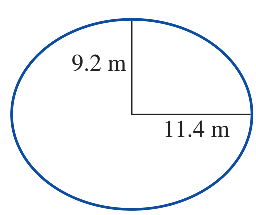


4 Calculate the area of each of the ellipses below, correct to 1 decimal place.

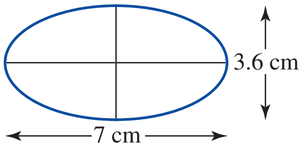
a



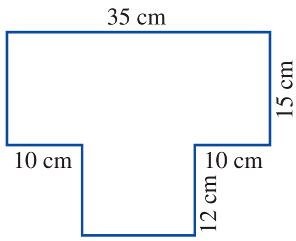
b



c

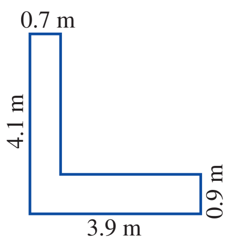


5 Calculate the area of the figure below.

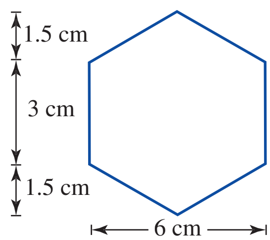


6 Calculate the area of each of the figures below. Where appropriate, give your answer correct to 2 decimal places.

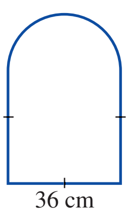
a



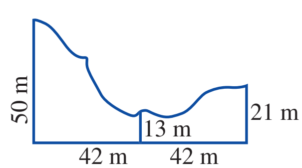
b



c

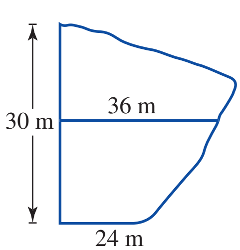


7 Use Simpson's rule to approximate the area below.

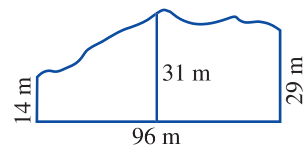


8 Use Simpson's rule to find an approximation for each of the areas below.

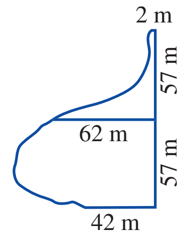
a



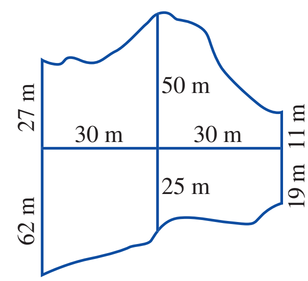
b



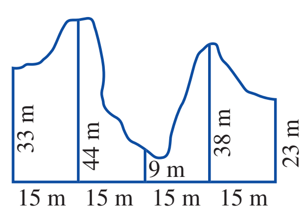
c



9 By dividing the area shown below into two sections, use Simpson's rule to find an approximation for the area.

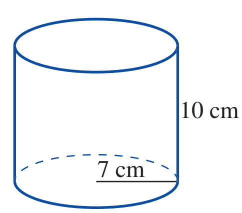


10 Use Simpson's rule twice to find an approximation for the area below.

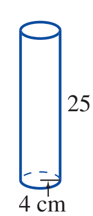


11 Calculate the surface area of each of the closed cylinders drawn below, correct to 1 decimal place.

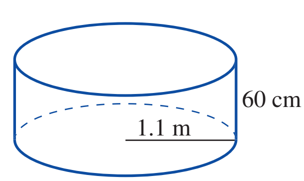
a



b



c



12 Calculate the surface area of an open cylinder with a diameter of 9 cm and a height of 15 cm. Give your answer correct to the nearest whole number.

13 Calculate the surface area of a sphere with:

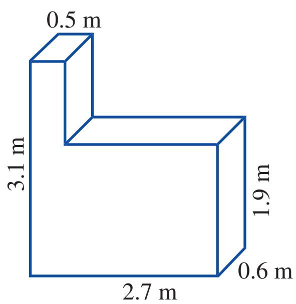
a a radius of 5 cm

b a radius of 2.4 m

c a diameter of 156 mm.

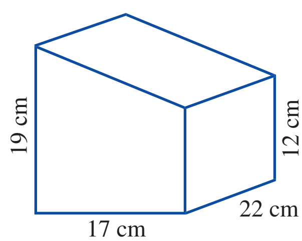
Give each answer correct to the nearest whole number.

14 Calculate the volume of the solid drawn below.

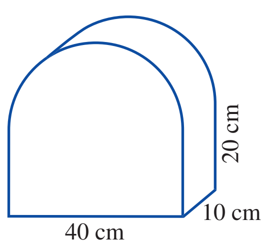


15 Calculate the volume of each of the solids drawn below. Where necessary, give your answer correct to the nearest whole number.

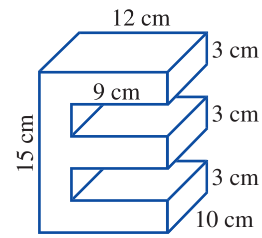
a



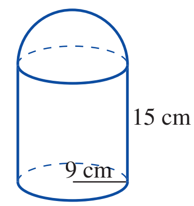
b



c



16 Calculate the volume of the figure drawn on the right, correct to 2 decimal places.



17 A sphere has a diameter of 16 cm when measured to the nearest centimetre.

a State the maximum error made in the measurement of the radius.

b Calculate the volume of the sphere. Answer correct to the nearest whole number.

c Calculate the maximum percentage error in the volume of the sphere.

18 An aluminium soft drink can has a diameter of 8 cm and a height of 10 cm.

a Calculate the capacity of the can, in millilitres, correct to the nearest 10 millilitres.

b The machine that cuts the aluminium for the can is mistakenly set to 12 cm. Calculate the percentage error in the capacity of the can (correct to the nearest whole number).

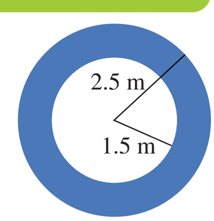
EXTENDED RESPONSE

1 The figure on the right shows a section of a concrete drainage pipe.

a Calculate the area of the annulus, correct to 1 decimal place.

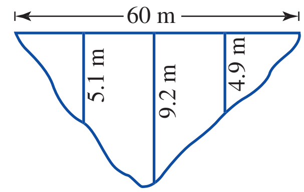
b Calculate the volume of concrete needed to make a 5 m length of this pipe (correct to 1 decimal place).

c Calculate the volume of water that will flow through the 5 m length of the pipe (in litres, to the nearest 100 L).



d Calculate the surface area of a 5 m section of pipe (correct to the nearest m^2). (*Hint:* Include the area of the inside of the pipe.)

2 The diagram on the right shows the cross-section of a river.



a Use two applications of Simpson's rule to find the approximate area of the river's cross-section.

b If the river flows with this cross-section for approximately 800 m, calculate the volume of the river.

c The length of the river has been approximated to the nearest 100 m. Calculate the maximum percentage error in calculating this volume.

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Digital doc
Test Yourself
doc-1319
Chapter 2

eBookplus ACTIVITIES

Are you ready?

Digital docs *(page 40)*
- SkillSHEET 2.1 (doc-1303): Area of a circle.
- SkillSHEET 2.2 (doc-1304): Areas of squares, rectangles and triangles.
- SkillSHEET 2.6 (doc-1312): Volume of cubes and rectangular prisms.
- SkillSHEET 2.7 (doc-1313): Volume of triangular prisms.
- SkillSHEET 2.8 (doc-1314): Volume of cylinders.
- SkillSHEET 2.9 (doc-1315): Volume of a sphere.
- SkillSHEET 2.10 (doc-1316): Volume of a pyramid.
- SkillSHEET 2.11 (doc-1317): Error in linear measurement.

2A Area of parts of the circle

Tutorial
- WE3 int-2411: Learn how to calculate the area of an annulus. *(page 42)*

Digital docs
- SkillSHEET 2.1 (doc-1303): Area of a circle. *(page 43)*

2B Area of composite shapes

Tutorial
- WE6 int-2412: Learn how to calculate the area of a composite shape. *(page 47)*

Digital docs
- SkillSHEET 2.2 (doc-1304): Areas of squares, rectangles and triangles. *(page 48)*
- SkillSHEET 2.3 (doc-1305): Using Pythagoras' theorem. *(page 49)*
- Spreadsheet: (doc-1306): Pythagoras. *(page 49)*
- GC program — Casio (doc-1307): Mensuration. *(page 49)*

2C Simpson's rule

Tutorial
- WE10 int-2413: Learn how to apply Simpson's rule. *(page 54)*

Digital docs
- SkillSHEET 2.4 (doc-1308): Substitution into formulas. *(page 55)*
- WorkSHEET 2.1 (doc-1309): Perform calculations involving area. *(page 57)*

2D Surface area of cylinders and spheres

Digital docs
- SkillSHEET 2.5 (doc-1310): Circumference of a circle. *(page 60)*
- Spreadsheet (doc-1311): Volume. *(page 63)*
- Investigation (doc-2747): Packaging. *(page 63)*

2E Volume of composite solids

Digital docs
- SkillSHEET 2.6 (doc-1312): Volume of cubes and rectangular prisms. *(page 66)*
- SkillSHEET 2.7 (doc-1313): Volume of triangular prisms. *(page 67)*
- SkillSHEET 2.8 (doc-1314): Volume of cylinders. *(page 67)*
- SkillSHEET 2.9 (doc-1315): Volume of a sphere. *(page 68)*
- SkillSHEET 2.10 (doc-1316): Volume of a pyramid. *(page 69)*
- Investigation: (doc-2748): Maximising volume. *(page 70)*

2F Error in measurement

Tutorial
- WE18 int-2414: Learn how to calculate percentage error. *(page 71)*

Digital docs
- SkillSHEET 2.11 (doc-1317): Error in linear measurement. *(page 72)*
- WorkSHEET 2.2 (doc-1318): Perform calculations involving volume. *(page 73)*

Chapter review

Digital doc
- Test Yourself (doc-1319): Take the end-of-chapter test to test your progress. *(page 79)*

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